TPP Mark 2013
September 19, 2013

The problem we present this year concerns the impossibility of the 14-15 puzzle of Sam Loyd.

1. Outline
The 14-15 puzzle of Sam Loyd states that if you switch
the positions of the #14 and #15 pieces in a 15-puzzle
(or 15-game), there is no combination of piece moves
that will solve it.

Reference：<Wikipedia>

2. Problem
Using the theorem proving system of your choice,
please prove that the 14-15 puzzle is unsolvable.

※ Notes
・ The initial state of the puzzle is that in which each piece
  is in its correct position except for #14 and #15 whose positions
  are switched.
・ The movement of the pieces {1, 2,..., 16} (16 is the empty square)
  can be expressed as interchanges or transpositions between two pieces.
  Please formally define these movements.

※ Postscript
・ To reduce the scale of the problem, you may consider the 8-puzzle
  version instead ("brute force" solutions are highly welcome).

3. Reference
’15puzzle no Suuri '<http://www.tcp-ip.or.jp/~n01/math/combinatorics/15puzzle/15puzzle.pdf >